The Admissibility Equation: A Mathematical Theory of Commitment

Abstract

This paper formalizes a mathematical theory of commitment derived from Procurement Physics. It introduces the **Admissibility Equation**, a necessary and sufficient condition governing whether irreversible commitments made under uncertainty can remain stable. The framework integrates three invariant constraints—learning (feasibility), liability alignment (singularity), and system capacity—into a single inequality that defines the boundary between admissible and inadmissible commitments. When violated, failure is not probabilistic or managerial but structurally locked in at the moment of commitment.

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1. Introduction

Large coordinated undertakings routinely fail in predictable ways. Conventional explanations attribute these outcomes to poor management, misaligned incentives, or execution error. This paper advances a different claim: that failure is determined earlier, at discrete commitment points, by violations of a small set of structural constraints.

The purpose of this paper is to present the mathematical core of that claim. Rather than proposing best practices or optimization techniques, it defines a physical admissibility condition that governs whether commitments can be sustained once made.

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2. Commitment as a Discrete Event

Let k denote a **binding commitment point**. A commitment point is any moment at which optionality is irreversibly reduced, including but not limited to:

- fixing price,
- transferring risk,
- guaranteeing performance,
- locking schedule, or
- foreclosing design or delivery alternatives.

Commitment points are discrete. The system does not gradually commit; it crosses thresholds. The admissibility of the system must therefore be evaluated at each k.

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3. The Admissibility Equation

A delivery system is physically admissible if and only if, for every binding commitment point k:

$$F_k \cdot L_{a(k)} \geq \frac{\|C_k\|}{\|K_k\|}$$

If this inequality is violated at any k, failure energy is irreversibly stored in the system.

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4. Definition of Terms

4.1 Feasibility $F_k \in [0, 1]$

F_k represents the fraction of uncertainty collapsed prior to commitment. It is a measure of learning completeness.

- $F_k = 0$: guesswork; no validated feasibility
- $F_k = 1$: proven capability

This term embeds the **Law of Sequence**: learning must precede commitment.

4.2 Liability Alignment $L_{a(k)} \in [0, 1]$

$L_{a(k)}$ represents the fraction of downside liability borne by the actor with final authority at commitment point k.

- $L = 1$: authority and liability are singular
- $L < 1$: consequences are externalized

This term enforces the **Law of Singularity** mathematically.

4.3 Committed Constraint Vector C_k

C_k represents the severity of what is being promised:

$$C_k = (C_c, C_t, C_p)$$

where: - C_c = cost tightness, - C_t = schedule compression, - C_p = performance rigidity.

The magnitude $\|C_k\|$ measures how unforgiving the commitment is.

4.4 System Capacity Vector K_k

K_k represents demonstrated capacity to absorb load:

$$K_k = (K_c, K_t, K_p)$$

where each component corresponds to the system's proven ability to absorb stress in cost, time, and performance.

The magnitude $\|K_k\|$ measures survivable stress.

This term normalizes the **Law of Capacity**.

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5. Interpretation of the Inequality

5.1 Left-Hand Side: Reality

The product $F_k \cdot L_{a(k)}$ represents reality-constrained behavior.

- Learning without liability produces optimism.
- Liability without learning produces fear pricing.
- Only their product stabilizes decision-making.

5.2 Right-Hand Side: Structural Demand

The ratio $\|C_k\|/\|K_k\|$ represents structural stress imposed by the commitment.

- Small promises relative to capacity yield stability.
- Large promises relative to capacity are physically impossible.

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6. Failure as Stored Energy

When the admissibility condition is violated, the system does not fail immediately. Instead, it stores failure energy that is later released as:

- cost overruns,
- schedule collapse,
- claims and disputes,
- scope erosion, or
- performance degradation.

Downstream management effort may delay manifestation but cannot eliminate the stored energy.

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7. Consequences for System Design

The admissibility equation reframes governance and delivery design:

- Risk cannot be transferred faster than feasibility is established.
- Authority without liability is mathematically destabilizing.
- Oversight that reduces capacity worsens admissibility.

The equation does not prescribe behavior; it defines what is structurally possible.

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8. Conclusion

The Admissibility Equation defines a law of motion for coordinated human systems under uncertainty. It explains why failure patterns recur across domains and why post-hoc explanations consistently misdiagnose root causes.

Once commitments are made in violation of admissibility, outcomes are no longer contingent on intent, effort, or competence. They are determined.

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